FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – VII
PAPER –2
TEST DATE: 28-04-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A

Sol. $\frac{dP}{dh} = -\rho g$, $B = \frac{-dP}{dV/V}$ and $\frac{dV}{V} = -\frac{d\rho}{\rho}$ (as M = Vρ)

Above three relations gives

$$B\int_{\rho_0}^{\rho}\frac{d\rho}{\rho^2} = g\int_0^h dh$$

$$\Rightarrow \quad \frac{1}{\rho_0} - \frac{1}{\rho} = \frac{gh}{B} \Rightarrow 1 - \frac{\rho_0}{\rho} = \frac{\rho_0 gh}{B} \Rightarrow \frac{\rho_0}{\rho} = 1 - \frac{\rho_0 gh}{B} \quad ...(1)$$

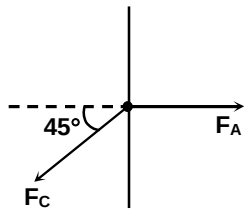
$$\Rightarrow \int_{P_0}^{P} dP = B\int_{\rho_0}^{\rho}\frac{d\rho}{\rho}$$

$\Rightarrow$ $P = P_0 + B \ln(\rho/\rho_0)$ (using eq. (1))

$$\Rightarrow P = P_0 + B\ln\left(1 - \frac{\rho_0 gh}{B}\right)^{-1} = P_0 - B\ln\left(1 - \frac{\rho_0 gh}{B}\right)$$

2. A

Sol. When $F_A = \frac{F_c}{\sqrt{2}}$, resultant force will be downward and so the surface will be flat.



3. A

Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v}-\frac{2}{-15}=\frac{-1}{-10}\Rightarrow v=-30cm \text{ and } \frac{4/3}{v'}-\frac{2}{-5}=\frac{4/3-2}{\infty}\Rightarrow v'=\frac{-10}{3}cm$$

4. C

Sol. $u=\sqrt{5g_{eff}\ell}=\sqrt{50}=5\sqrt{2}m/s$ and $T-mg\cos\theta=\frac{mu^2}{\ell}\Rightarrow T=60N$

5. ABC

Sol. $V = E_1d_1 + E_2d_2$ ...(1)

$J = \sigma_1E_1 = \sigma_2E_2$ ...(2)

$E_1 = E_0/\varepsilon_1$ and $E_2 = E_0/\varepsilon_2$ ...(3)

From (2) and (3)

$$\frac{\sigma_1}{\sigma_2}=\frac{\varepsilon_1}{\varepsilon_2}$$

From equation (1) and (2) $E_1 = \frac{V\sigma_2}{d_1\sigma_2+d_2\sigma_1}$

Now field inside first dielectric

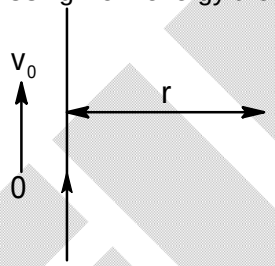
$$E_1=\frac{q/A}{\varepsilon_0}-\frac{q/A}{2\varepsilon_0}\left(1-\frac{1}{\varepsilon_1}\right)-\frac{q/A}{2\varepsilon_0}\left(1-\frac{1}{\varepsilon_2}\right)+\frac{\sigma}{2\varepsilon_0}=\frac{\varepsilon_1E_1}{2}\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right)+\frac{\sigma}{2\varepsilon_0}$$

$$\frac{V\sigma_2}{d_1\sigma_2+d_2\sigma_1}-\frac{\varepsilon_1}{2}\frac{V\sigma_2}{(d_1\sigma_2+d_2\sigma_1)}\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right)=\frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow\quad \sigma=\frac{V\sigma_2\varepsilon_0}{d_1\sigma_2+d_2\sigma_1}\left(1-\frac{\sigma_1}{\sigma_2}\right)=\frac{V\varepsilon_0(\sigma_2-\sigma_1)}{(d_1\sigma_2+d_2\sigma_1)}$$

6. AD

Sol. Using work energy theorem from the frame of an observer O moving parallel to the wire.



$$q_0E=q_0v_0\frac{\mu_0I_0}{2\pi r}$$

$$\int_{r_0}^{r_{max}}qv_0\frac{\mu_0I_0}{2\pi r}dr=0-\frac{1}{2}m\left(v_0\sqrt{2}\right)^2$$

$$\frac{qv_0\mu_0I_0}{2\pi}\ln\frac{r_{max}}{r_0}=-mv_0^2$$

$$r_{max}=r_0e^{\frac{-2\pi mv_0}{\in\mu_0I_0}}$$

7. ABD

Sol. $\frac{I_t}{I_i}=\frac{A_t^2\rho_2}{A_i^2\rho_1}\frac{v_2}{v_1}=\left(\frac{4}{3}\right)^2\frac{1}{4}(2)=\frac{8}{9}$

Where $\frac{A_t}{A_i}=\frac{2v_2}{v_1+v_2}$ and $\frac{v_2}{v_1}=\sqrt{\frac{\rho_1}{\rho_2}}=\frac{1}{2}$

$A_r=A_i\left(\frac{v_2-v_1}{v_1+v_2}\right)$ as $v_1>v_2$ for $\frac{\rho_2}{\rho_1}=4$ so reflected wave suffers a phase change of π.

SECTION – B

8. 10

Sol. $\frac{Ke^2}{r^2}=\frac{mv^2}{r/2}$ ... (1)

$mvr=\frac{nh}{2\pi}$ ... (2)

Solving (1) and (2)

$r=\frac{n^2h^2}{2\pi^2mKe^2}$

$r = 2n^2r_0$ (where r_0 = 0.53 Å and n = 3)

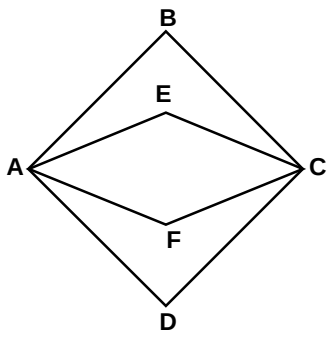
r = 9.54Å

9. 2

Sol. As the square BEDF lies on plane of symmetry so it can be removed.

So the circuit after its removal is as shown

$R_{eq}=\frac{R}{2}=0.5\Omega$



10. 2

Sol. $y = y_1 + y_2 + y_2$

$\frac{1}{k_{eq}}=\frac{1}{4k}+\frac{1}{16k}+\frac{1}{64k}\Rightarrow k_{eq}=\frac{64k}{21}$

$T=2\pi\sqrt{\frac{m}{k_{eq}}}$ = 0.25 sec

11. 7

Sol. $ms(T_1-T_0)\times\frac{30}{100}=ms(T_1-T_2)$

$\Rightarrow T_2=0.7T_1+0.3T_0$ .....(1)

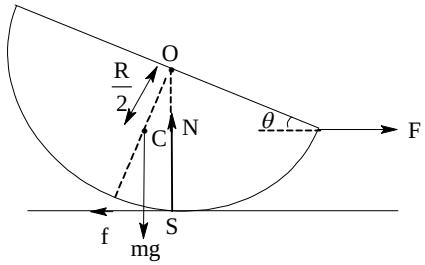
$t=\frac{1}{K}\ln\frac{T_1-T_0}{T_2-T_0}$ .....(2)

Substituting the value of T_2 in equation (2)

$$t = \frac{1}{K}\ln\frac{10}{7}$$

12. 6

Sol.



$F = f$ ...(1)

$N = mg$ ...(2)

$\sum \vec{T} = 0$ (about point S)

$$FR(1-\sin\theta) = mg\frac{R}{2}\sin\theta$$

$$\mu mgR(1-\sin\theta) = mg\frac{R}{2}\sin\theta$$

$$\Rightarrow \sin\theta\frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

13. 3

Sol. $$\frac{-(\Delta T/\Delta t)_s}{-(\Delta T/\Delta t)_c} = \frac{eA_s\sigma T^4/m_s s}{eA_c\sigma T^4/m_c s} = \left(\frac{A_s}{A_c}\right)\left(\frac{m_c}{m_s}\right) = \frac{4\pi R^2}{6a^2}\left(\frac{2}{9\pi}\right) \text{ and } m_s = \frac{9\pi}{2}m_c \Rightarrow \frac{R}{a} = \frac{3}{2}$$

$$\frac{-(\Delta T/\Delta t)_c}{-(\Delta T/\Delta t)_s} = 3$$

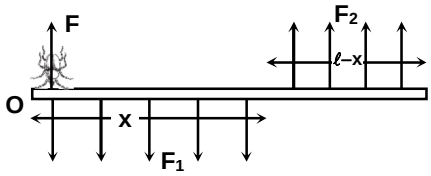
SECTION – C

14. 1.41

15. 2.05

Sol. (Q.14-15).

$F = F_1 - F_2$ ...(1)



$F_1 = \mu\frac{m}{\ell}xg$ and $F_2 = \mu\frac{m}{\ell}(\ell - x)g$

Gives $\frac{F_1}{F_2} = \frac{x}{\ell - x}$ ...(2)

Torque about point 'O'

$$F_1\frac{x}{2} = F_2\left(\frac{\ell + x}{2}\right)$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{\ell + x}{x}$$ ...(3)

Solving (2) and (3)

Gives x = $\ell/\sqrt{2}$ So, x = $\frac{2}{\sqrt{2}} = 1.41\text{m}$ and $F_1 = \frac{\mu mg}{\sqrt{2}}$, $F_2 = \mu\frac{\left(\sqrt{2}-1\right)mg}{\sqrt{2}}$

So, $F = F_1 - F_2 = \mu mg\left(\sqrt{2}-1\right)$ = 2.05 mN

16. 0.22

Sol. $\vec{F} = \frac{I\mu_0 I_0}{2\pi\ell}3\ell(-\hat{i}) + \int_{\ell}^{2\ell} I\left(\frac{\mu_0 I_0}{2\pi r}\right)\frac{dr}{\cos\theta}\sin\theta\times 2\left(\hat{i}\right) + \frac{I\mu_0 I_0 \ell\hat{i}}{2\pi(2\ell)} = 2.2\times10^{-7}\left(-\hat{i}\right)$ = 0.22 μN

17. 0.22

Sol. $e = \frac{\mu_0 I_0 v 3\ell}{2\pi\ell} - \int_{\ell}^{2\ell}\frac{\mu_0 I_0 v}{2\pi r}\frac{dr}{\cos\theta}\sin\theta\times 2 - \frac{\mu_0 I_0 v\ell}{2\pi(2\ell)}$ = 2.2 × 10^{-7} volts = 0.22 μV

Chemistry

PART – II

SECTION – A

18. C
Sol. Free radical shows resonance

19. D
Sol. Seliwanoff's test is used to distinguish aldose and ketose.
Halogen and hydroxy group give Victor Mayer's test.

20. C
Sol. $BaCrO_4$ is Yellow in color, Q= NH_3

21. C
Sol. $2HClO_3+H_2C_2O_4 \longrightarrow 2ClO_2+2CO_2+2H_2O$
$2ClO_2+2O_3 \rightarrow Cl_2O_6+2O_2$

22. B, D
Sol. bis(cyclopentadienyl) bis (dinitrogen) titanium(II)
De to back bonding, electrons enters into ABMO of N_2 so bond length of N-N bond increases

23. BCD
Sol. If we do the complete removal of impurities, then sol coagulates.

24. A, B, C
Sol $\Delta S_{system} = -\frac{0.5\text{mol}\times 40.6\text{Kj/mol}}{373\text{K}} = -54.42\ J/K$
$\text{dG=dH-d}(\text{TS})$
$= d(U+PV)-d(TS)$
$\Delta G_{P,T} = dU + Pdv - TdS$
$= dq + dw_{rev} + dw_{irr} + PdV - TdS$
$\Delta\text{G=dw}_{\text{non mechanical on the system}}$
$\text{dw}_{\text{non mechanical by the system}} = -\Delta G_{T,P}$
So D is true.
$dG = dH - d(TS)$
$= d(V+PV)-d(TS)$
$= dU + PdV + VdP - TdS - SdT$
$= dq + \text{dW}_{\text{mech}} + \text{dW}_{\text{non mech.}} + PdV + VdP - TdS - SdT$
$\text{dG=dW}_{\text{non mech.}} + \text{VdP-SdT}$
So, 'C' is false
B is false
For part "A" , $\Delta S_{system} = -54.42\ J/K$

SECTION – B

25. 5

Sol. $\frac{(x+y)}{Z}=\frac{(10+0)}{2}=5$

26. 6

Sol. Zn, Fe, Pb, Cu, Ag, Sn

27. 5

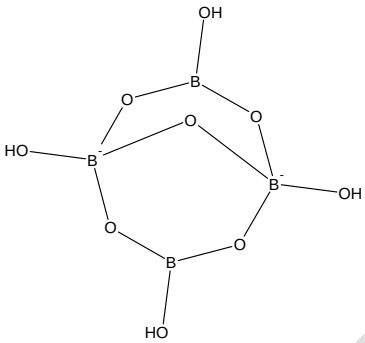
Sol. Sb, Se, Te, Ag, Pt

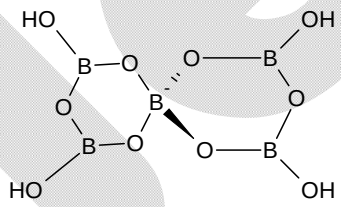
28. 4

Sol. i, ii, iii,vi

29. 11

Sol.





5 +6=11

30. 7

Sol. (i) $Zn+\text{dil. } H_2SO_4 \rightarrow ZnSO_4+H_2$

(ii) $H_2C_2O_4+\text{conc. } H_2SO_4 \rightarrow CO{+}CO_2$

(iii) $P_4+4SOCl_2 \rightarrow 4PCl_3+S_2Cl_2+2SO_2$

(iv) $NH_3+O_2 \xrightarrow{Pt} NO+H_2O$

(v) $H_3PO_3 \xrightarrow{\Delta} PH_3+H_3PO_4$

(vi) $Fe^{3+}+SO_2 \rightarrow Fe^{2+}+SO_4^{2-}$

(vii) $NaCl+H_2SO_4 \rightarrow HCl+NaHSO_4$

(viii) $4HCl+O_2 \xrightarrow{CuCl_2} 2Cl_2+2H_2O$

SECTION – C

31. 64.00

32. 0.15

Sol. (Q. 31 – 32)

$$[Br]^- = [Ag(NH_3)_2]^+ \quad + \quad [Ag^+]$$

$$\frac{K_{sp}}{[Ag^+]} = [Ag^+] \times [NH_3]^2 \times 8 \times 10^6 + \quad [Ag^+]$$

$$\frac{K_{sp}}{[NH_3]^2 \times 8 \times 10^6 + 1} = [Ag^+]^2 \Rightarrow [Ag^+]^2 = \frac{2 \times 10^{-10}}{0.04 \times 8 \times 10^6 + 1} \approx \frac{2 \times 10^{-10}}{32 \times 10^4} = \frac{1}{16} \times 10^{-14}$$

$$[Ag^+] = \frac{1}{4} \times 10^{-7}, \quad [Br^-] = 8 \times 10^{-3}$$

$$\Lambda_{m,\,Br^-} \approx \Lambda^0_{m,\,Br^-} = \frac{\kappa}{1000 \times [Br^-]} \Rightarrow 8\text{mS m}^2/\text{mol} = \frac{\kappa}{1000 \times 8 \times 10^{-3}} \Rightarrow \kappa = 64 m\text{S/m}$$

$$\Lambda_{m,\,Ag^+} \approx \Lambda^0_{m,\,Ag^+} = \frac{\kappa}{1000 \times [Ag^+]} \Rightarrow 6\text{mS m}^2/\text{mol} = \frac{\kappa}{1000 \times 0.25 \times 10^{-7}} \Rightarrow \kappa = 150 \times 10^{-6} m\text{S/m} = 0.15 \mu\text{S/m}$$

33. 19.76

34. 11.00

Sol. (2+ 2) addition

Mathematics

PART – III

SECTION – A

35. D

Sol. Apparently $p + q = -1$, $pq = 2$, $p^2 + q^2 + 2pq = 1$, $p^2 + q^2 = -3$

$f(x)g(x) = x^6 - (p + q)x^5 + (p + q + pq)x^4 - (p^2 + q^2 + 2)x^3 + (p + q + pq)x^2 - (p + q)x + 1$

Hence $f(x)g(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = \dfrac{x^7 - 1}{(x-1)}$

$$\frac{d^2}{dx^2}\left(f(x)g(x)\right) = 18$$

36. C

Sol. Apparently $\vec{P}_k = a\sin\left(\dfrac{2\pi k}{N}\right)\hat{i} + b\cos\left(\dfrac{2\pi k}{N}\right)\hat{j}$

Hence, $\lim\limits_{N\to\infty}\sum\limits_{k=0}^{N-1}\left[\vec{P}_k \quad \vec{P}_{k+1} \quad \vec{P}_k \times \vec{P}_{k+1}\right] = 4\pi^2a^2b^2$

37. B

Sol. Required number of numbers = ${}^nC_0\,(2)^n + {}^nC_2\,(2)^{n-2} + \ldots$

We know that

$(1 + x)^n + (x - 1)^n = 2\sum\limits_{k=0}^{n} {}^nC_k x^k$

Hence put $x = 2$, get the result

$2\,[{}^nC_0\,(2)^n + {}^nC_2\,(2)^{n-2} + {}^nC_4\,(2)^{n-4} + {}^nC_6\,(2)^{n-6} + \ldots]$.

38. B

Sol. Clearly $\tan\beta \in Q$ if $p^2 + q^2$ is square of an integer.

39. A, D

Sol. Circumcircle of ΔABC passes through the focii $\left(\dfrac{1}{4k+4} - 3, 0\right)$ of the given parabola.

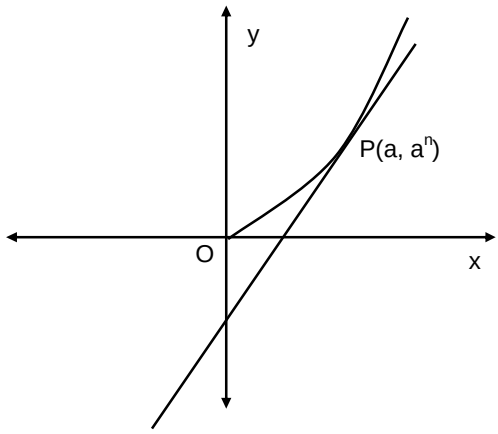
40. A, D

Sol. Slope of normal at $P(a, a^n)$ is $\dfrac{-1}{na^{n-1}}$.

Equation of normal is $(y - a^n) = \dfrac{-1}{na^{n-1}}(x - a)$

Put $x = 0$ to get y-intercept as $b = a^n + \dfrac{1}{na^{n-2}}$

$$\Rightarrow \lim_{a\to 0} b = \begin{cases} 0, & \text{if } n < \dfrac{1}{2} \\ \dfrac{1}{2}, & \text{if } n = 2 \\ \infty, & \text{if } n > 2 \end{cases}$$



41. A, D

Sol. Let O be the intersection of diagonals AC and BD.

Let position vector of $A(\vec{a})$, $B(\vec{b})$, $C(\vec{c})$, $D(\vec{d})$. Let position vector of G be $(\vec{g})$.

Let $\frac{AM}{AB} = m$, $\frac{BN}{BC} = n$, $\frac{DP}{DC} = p$

Now, $\overrightarrow{OM} = (1-m)\vec{a} + n\vec{b}$

$\overrightarrow{ON} = (1-n)\vec{b} + n\vec{c}$

$\overrightarrow{OP} = (1-p)\vec{d} + p\vec{c}$

$\Rightarrow \vec{g} = \frac{(1-m)\vec{a} + n\vec{b} + (1-n)\vec{b} + n\vec{c} + (1-p)\vec{d} + p\vec{c}}{3}$ as $\vec{a} = -\vec{c}$ and $\vec{d} = -\vec{b}$

$\vec{g} = (m+n+p-1)\vec{c} + (m-n+p)\vec{b}$

If $G(\vec{g})$ is collinear with AC

$\Rightarrow m - n + p = 0$

If $G(\vec{g})$ is collinear with BD

$\Rightarrow m + n + p - 1 = 0$.

SECTION – B

42. 8317

Sol. Suppose $S = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \frac{mnk\left(3m^2n + mnk\right)5^{-(m+n+k)}}{(m+n)(n+k)(k+m)}$

Apparently, the series remains unchanged by interchanging

$m \to n,\ n \to k,\ k \to m,\ m \to k,\ k \to n,\ n \to m$

Hence $6S = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \frac{mnk\left(3\left(m^2n + n^2k + k^2n + n^2m + k^2m + m^2n\right)\right)}{5^{m+n+k}(m+n)(n+k)(k+m)}$

$2S = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \frac{mnk}{5^{m+n+k}} \frac{(m+n)(n+k)(k+m)}{(m+n)(n+k)(k+m)}$

$S = \frac{1}{2}\sum_{m=1}^{\infty}\left(\frac{m}{5^m}\right)\sum_{n=1}^{\infty}\left(\frac{n}{5^n}\right)\sum_{k=1}^{\infty}\frac{k}{5^k}$

$S = \frac{1}{2}\left(\frac{5}{16}\right)^3 = \frac{125}{8192}$

43. 4

Sol. $-1 \le m^2 - 3m + 1 \le 1$

$m^2 - 3m + 2 \ge 0 \Rightarrow m \in (-\infty, 1] \cup [2, \infty)$

$m^2 - 3m \le 0 \Rightarrow m \in [0, 3]$

$\Rightarrow m \in [0, 1] \cup [2, 3]$

$\sec\frac{|x|}{3}$ have period 6π.

If $m < 4$, $[e^{m-4}] = 0$

If $m^2 - 3m + 1 = 0$ then $m = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ which are both less than 4

44. 1010

Sol. Apparently $P_k = 2^k$

Suppose $\text{matrix} B = (w) X = \begin{bmatrix} w & 0 \\ 2w - w^2 & w^2 \end{bmatrix}$

$B^2 + B + I = O \;\&\; B^3 + B^2 + B = O \Rightarrow B^3 = I$

$B^4 = B$

Hence, $B^8 = B^2$

$B^{16} = B^4 = B$

If $Y = \sum_{k=1}^{2020} [B]^{2^k}$ then $|Y| = (1010)^2$

45. 3

Sol. $\omega = e^{i2\pi/28}$

$x = \frac{1}{2}|\omega - \bar{\omega}| = \frac{1}{2}\left|2i\sin\frac{\pi}{14}\right| = \sin\frac{\pi}{14}$

If $\theta = \frac{\pi}{14} \Rightarrow 3\theta + 4\theta = \frac{\pi}{2}$

$\text{Sin}3\theta = \cos 4\theta$

$3x - 4x^3 = 2\cos^2 2\theta - 1 = 2(1 - 2\sin^2\theta)^2 - 1$

$3x - 4x^3 = 2(1 - 2x^2)^2 - 1$

$= 2(4x^4 - 4x^2 + 1) - 1$

$= 8x^4 - 8x^2 + 1$

$8x^4 + 4x^3 - 8x^2 - 3x + 1 = 0.$

46. 3960

Sol. Two 'a' can be filled in ${}^4C_2 \cdot {}^4P_2 = 72$ ways.

Two 'b' can be filled in 72 ways

$\Rightarrow$ two 'a' and two 'b' can be filled in 72^2 ways.

Now, we have to exclude the cases when

(i) both 'a' and 'b' are in same cell i.e. 72 ways

(i) one 'a' and 'b' are in same cell i.e. 72×16 ways

Required number of ways = $72^2 - 72 - 72 \times 16 = 3960$.

47. 800

Sol. Using $\begin{vmatrix} 5 & a & b \\ 13 & p & q \\ 15 & m & n \end{vmatrix} = \begin{vmatrix} 2 & a & b \\ 4 & p & q \\ 9 & m & n \end{vmatrix} + \begin{vmatrix} 3 & a & b \\ 9 & p & q \\ 6 & m & n \end{vmatrix} = 800$

SECTION – C

48. 2211.84

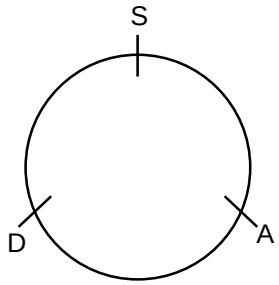
49. 1.50

Sol. Possible pairs of (m, n, p, q) are.

(2, 0, –5, 7) and (–2, 0, –7, 5)

50. 33120.00

Sol.



No. of ways in which seating arrangements can be done is $\underline{|8} - 10\underline{|6} = 33120$

51. 00042.14

Sol. $F_n = 5^n + 5(-1)^n$

$F_{11} = 5^{11} - 5$

Probability = $\frac{5^{11}-5}{6^{11}}$, hence m = n = 5, p = 6

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